Derived Geometry, Symplectic Geometry, and Representation Theory

December 10–12, 2019

Kick-off meeting of the ERC project DerSympApp

Unless otherwise specified, talks are taking place in room 330.

SCHEDULE

	Tuesday	**Wednesday**	**Thursday**
9 : 00	*welcome coffee*	———	———
9 : 30	BRANTNER	MANN	———
10 : 00			STEFFENS I
10 : 30	*coffee break*	*coffee break*	
11 : 00	LERAY	KELLER	*coffee break*
11 : 30			MINETS
12 : 00	*lunch break*	*lunch break*	(AGATA seminar)
12 : 30			**room 430**
13 : 00			*lunch break*
13 : 30			*(common room)*
14 : 00	REMBADO	HOSGOOD	
14 : 30			STEFFENS II
15 : 00	*coffee break*	*coffee break*	
15 : 30	BOZEC	———	*coffee break*
16 : 00		———	———
16 : 30	———	———	———

ABSTRACTS

Tristan Bozec

Relative critical loci and moduli stacks of quiver representations

Inspired by some classical considerations in symplectic geometry, one can consider the critical locus of a function relative to some submersion. Precisely, in the derived setting, it may be interesting to consider the pull-back of the graph of a closed 1-form on some stack X along the Lagrangian correspondence $T^*B \leftarrow \pi^*T^*B \to T^*X$ induced by a morphism $\pi \to B$. When π is the pull-back of an inclusion of quivers and the 1-form is the differential of some potential, we obtain the category of perfect modules over some generalized "relative" version of Ginzburg's dg-algebra, which is 3-Calabi-Yau. If time permits we will see one (non-derived) example of a Lagrangian subvariety thus obtained.

This is a joint work with D. Calaque and S. Scherotzke.

Lukas Brantner

Deformation theory and Lie algebras in characteristic p

The infinitesimal structure of any moduli space in characteristic zero is controlled by a differential graded Lie algebra. This far-reaching generalisation of Kodaira–Spencer theory was established by Lurie and Pridham, based on previous work of Deligne, Drinfeld, Feigin, Hinich, and others. In this talk, I will explain how to generalise the above statement to finite and mixed characteristic.

This is joint work with Akhil Mathew.

Timothy Hosgood

Simplicial Chern–Weil theory

Characteristic classes of principal bundles can be well understood by studying the cohomology of the corresponding classifying space; there is a similar story for locally free sheaves (a more algebraic way of looking at GL-principal bundles) that uses the theory of connections, their curvatures, and invariant polynomials. This so-called Chern–Weil theory has been studied by differential geometers, but can be adapted to work in the complex-analytic case as well. In this talk we will try to understand the role of connections as obstruction classes; what the 'good' notion of a simplicial sheaf is (for our purposes); and maybe how these objects give us a much simpler way of understanding coherent analytic sheaves.

Corina Keller

Smooth 1-dimensional field theories over a manifold

An axiomatic approach to topological field theory (TFT) has been developed in the late 80's by Atiyah and Segal, defining a functorial TFT as a symmetric monoidal functor from a topological bordism category to a category of algebraic nature. Classification of such functorial TFTs has been conjectured by Baez and Dolan, in what is known as the cobordism hypothesis, and later proven by Lurie, who established a higher categorical equivalence between the space of fully extended n-dimensional TFTs and the underlying ∞-groupoid of fully dualizable objects in the target category. A natural extension is to study field theories where bordisms have geometric structure. A first step in this direction is to endow the bordism category with a smooth map to a fixed base manifold. In my talk I will introduce the notion of such smooth field theories over a manifold in the setting of higher category theory. I will then discuss their classification in dimension 1 and propose that one can characterize the space of all 1-dimensional smooth field theories over X as the space of certain representations of the path ∞-groupoid of X.

Johan Leray

Some properadic structures in geometry

The aim of this talk is to present how properads appear in (derived) geometry and how the properadic calculus can be useful in this framework. After recall the definition of properad, and how these objects encode algebraic structures (up to homotopy) with several inputs and several outputs, I will present two examples of such structures. The first is double Poisson algebras, which are the noncommutative analogous of Poisson algebras. The second is involutive BiLie algebras which are central in some conjectures made by Fukaya–Cieliebak–Latschev.

Etienne Mann

Gromov-Witten invariants of an hypersurface via derived algebraic geometry

Let E be a bundle over a smooth projective variety X with a section. Denote by Z the zero locus of this section. The question is the relation between the Gromov–Witten invariants of the hypersurface Z and the ones of X. This was done by Kim–Kresch–Pantev in genus 0. We will explain how to see this more naturally using derived algebraic geometry.

This is a work joint with Benjamin Hennion and Marco Robalo.

Valerio Melani (cancelled)

Tate geometry and (higher) affine Grassmannians

Motivated by a conjecture of Kapranov–Ginzburg–Vasserot, we study symplectic and Poisson structures on Tate stacks. After having introduced and discussed the conjecture, we will present our approach to Tate geometry; our formalism allows in particular to construct a (shifted) symplectic structure on the moduli space of perfect complexes over $Spec(k((t)))$. Moreover, we show how the local Hecke stack inherits a symplectic structure, and give a different perspective on the classical Poisson structure on the affine Grassmannian. We will also discuss how these results can hopefully be extended to higher dimensional situations, producing for example shifted Poisson structures on higher affine Grassmannians.

The talk is based on joint work in progress with Mauro Porta.

Alexandre Minets (AGATA seminar)

K-theoretic Hall algebras of surfaces and quantum toroidal $\mathfrak{gl}_n$

The elliptic Hall algebra, also known as quantum toroidal $\mathfrak{gl}_1$, appears in a variety of geometric contexts, ranging from skein algebras to Hilbert schemes of points. I will explain how some of these appearances can be generalized to the quantum toroidal $\mathfrak{gl}_n$. Special attention will be paid to its relation with K-theoretic Hall algebras of surfaces. If time permits, I will try to sketch how this perspective may lead to a refinement of mirror symmetry for Hitchin systems of Donagi and Pantev.

Gabriele Rembado

Symmetries of isomonodromy systems and quantisation of quiver varieties

We consider moduli spaces of meromorphic connections on the sphere, with simple poles in the complex plane and an irregular singularity at infinity. Varying the position of the simple poles and the coefficients at infinity in general modifies the extended monodromy/Stokes data of a connection; imposing that they stay fixed yields nonlinear differential equations (isomonodromy equations), which are in this case coded by an integrable Hamiltonian system (isomonodromy system). The classical systems at hand generalise Painlevé VI, V and IV, and have a natural $SL(2)$ group of symmetries which contains the Harnad duality.

Then in this talk we consider the quantum analogue of the previous statements. More precisely, we will explain how to deformation-quantise the moduli spaces of meromorphic connections, which is tantamount to quantise certain symplectic (Nakajima) quiver varieties, as well as the isomonodromy system. Then the $SL(2)$ symmetries may also be quantised to include the quantum/Howe duality, and the resulting quantum integrable system (which generalise the KZ, Casimir and dynamical connections) will be shown to be generically invariant.

Olivier Schiffmann (cancelled)

Pelle Steffens

Elliptic moduli problems in derived geometry

Consider a nonlinear elliptic PDE defined on the sections of a bundle over a compact manifold. Much of the power that theoretical physics brings to geometry comes from the study of the spaces of solutions (up to the action of some natural group of symmetries of the equation) of such geometric PDE's. Due to infinite dimensional transversality issues, these spaces are generally not manifolds, but instead glued together from singular zero loci of smooth functions, which are very hard to analyze "classically", that is, without the help of homotopy coherent mathematics.

In the first talk, I will explain the fundamentals of *derived C^∞ geometry*, which I will advertise as the "universal" solution to transversality problems in the context of higher algebra and higher category theory. In complete analogy with derived algebraic/spectral geometry, there is a robust theory of *derived manifolds* and more singular *derived geometric stacks*, whose deformation theory is well understood, and which can thus be equipped with meaningful geometric structures relevant to mathematical physics (i.e. orientations for enumerative invariant problems, or (shifted) symplectic structures for deformation quantization problems).

In the second talk, I will explain how moduli of elliptic PDE's ("elliptic moduli problems") fit naturally in the framework of derived C^∞ geometry. More precisely, we will see that an elliptic moduli problem admits a universal characterization as a (derived) stack, which avoids the problem of gluing singular spaces in the spirit of Grothendieck. Using techniques from global functional analysis, we will establish that an elliptic moduli problem is *representable* by a derived manifold, which makes the geometry of such spaces accessible.